Optimal loading plan for multiple trains in container terminals

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1 Problem description and formulation

The definition of the train loading plan is one of the decisional aspects to be dealt with in a container terminal [1]. In the literature, few research studies have been devoted to the train load planning problem; among them, in [2] three different integer linear programming formulations are proposed in which the real weight restrictions related to wagons configurations are considered, as in the present work. The model proposed in this paper is the extended version of the models described in [3]; the main new modelling aspects of this work in comparison with [3] refer to considering multiple trains, with different destinations, and taking into account the minimization of the distances between the stocking area and the train.

In the proposed model, the trains are supposed to be loaded by cranes sequentially and some rehandling operations in the stocking area are allowed. The decision problem can be defined as follows: given a set of containers characterized by different weights, lengths, commercial values and stowage positions in the yard and a set of trains composed by a set of wagons characterized by different lengths and weight restrictions, the problem is to determine how to assign containers to wagons in order to satisfy the physical constraints of wagons, while maximizing the train utilization and minimizing both the rehandling cost in the stoking area and the distances covered to transfer containers from the stocking area to the wagons.

The following notation is considered: $\mathcal{C}$ is the set of containers, ω_i is the weight of container i, π_i is the commercial value of container i, $\gamma_{ij} = 1$ indicates that container i is located below j, $\mathcal{T}$ is the (ordered) set of trains, $\mathcal{W}$ is the (ordered) set of wagons, $\mathcal{S}$ is the (ordered) set of slots, $\mathcal{S}_w \subset \mathcal{S}$ is the set of slots of wagon w, $\mathcal{S}_t \subset \mathcal{S}$ is the set of slots of train t, w_s indicates the wagon including slot s, $NS = |\mathcal{S}|$ is the total number of slots (i.e., the largest index associated with the slots), $\mathcal{B}_w$ is the set of weight configurations for wagon w, δ_{bs} is the maximum weight for slot s in the weight configuration b, d_{is} is

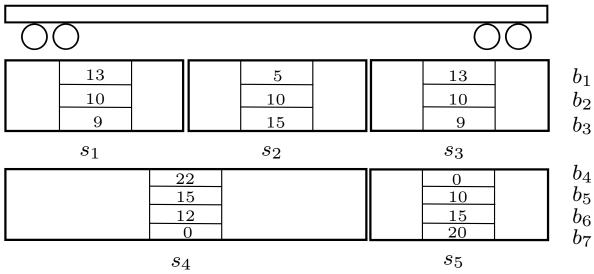

Figure 1: Sketch of wagon weight restriction.

the distance between container i and slot s, $\overline{\Omega}_w$ is the weight capacity of wagon w, $\overline{\Omega}_t$ is the total weight capacity of train t, α is the unitary rehandling cost, β is the coefficient weighting the cost term relevant to the distances in the cost function. In the notation it is assumed that the index associated with each slot univocally identifies it and that the slots indexes indicate the position of slots in the train; since the train is supposed to be loaded sequentially, slots with a smaller index are loaded before those with a higher index.

In order to clarify the notation regarding the wagon weight restrictions, consider the example sketch provided in Figure 1 in which a wagon with a capacity of 3 TEUs is shown; this car can be loaded either with three 20' containers or with one 20' container and one 40' container. Considering for example the first load configuration, there are 3 slots, i.e. s_1, s_2 and s_3, and three different weight configurations, b_1, b_2 and b_3, indicating the maximum weight per slot (for instance if the first weight configuration is chosen, the three slots can can be loaded with maximum weights $\delta_{b_1 s_1} = 13$, $\delta_{b_1 s_2} = 5$ and $\delta_{b_1 s_3} = 13$, respectively).

The problem decision variables are the following: $x_{is} \in \{0, 1\}$ is equal to 1 if container i is assigned to slot s; $y_{bw} \in \{0, 1\}$ is equal to 1 if weight configuration b is chosen for wagon w; $z_{ij} \in \{0, 1\}$, defined for all the pairs of containers (i, j) such that $\gamma_{ji} = 1$, is equal to 1 if container i is rehandled in order to load container j. Note that the x_{is} variables are defined only if the assignment of containers to slots is feasible and this can take into account different aspects; first of all, it is necessary to consider the fitting of the container length with the slot length, i.e. if the considered container is 40' long and the slot is 20', the corresponding variable is not defined. Moreover, given that trains with different destinations are considered, for each slot of a given train the x_{is} variables are defined only for the containers with the same destination of the train.

The train load planning problem can be stated with the following 0-1 linear programming formulation.

$$\min\ \alpha \cdot \sum_{i,j \in \mathcal{C}: \gamma_{ji}=1} z_{ij} + \beta \cdot \sum_{i \in \mathcal{C}} \sum_{s \in \mathcal{S}} d_{is} \cdot x_{is} + \sum_{i \in \mathcal{C}} \pi_i \cdot \left(1 - \sum_{s \in \mathcal{S}} x_{is} \right) \tag{1}$$

s.t.

$$\sum_{s \in \mathcal{S}} x_{is} \leq 1 \qquad \forall i \in \mathcal{C} \tag{2}$$

$$\sum_{i \in \mathcal{C}} x_{is} \leq 1 \qquad \forall s \in \mathcal{S} \tag{3}$$

$$\sum_{b \in \mathcal{B}_w} y_{bw} = 1 \qquad \forall w \in \mathcal{W} \tag{4}$$

$$\sum_{i \in \mathcal{C}} \omega_i \cdot x_{is} \leq \sum_{b \in \mathcal{B}_{w_s}} \delta_{bs} \cdot y_{bw} \qquad \forall s \in \mathcal{S} \tag{5}$$

$$\sum_{i \in \mathcal{C}} \sum_{s \in \mathcal{S}_w} \omega_i \cdot x_{is} \leq \overline{\Omega}_w \qquad \forall w \in \mathcal{W} \tag{6}$$

$$\sum_{i \in \mathcal{C}} \sum_{s \in \mathcal{S}_t} \omega_i \cdot x_{is} \leq \overline{\Omega}_t \qquad \forall t \in \mathcal{T} \tag{7}$$

$$\sum_{s \in \mathcal{S}} x_{js} \leq \sum_{s \in \mathcal{S}} x_{is} + z_{ij} \qquad \forall i, j \in \mathcal{C} : \gamma_{ji} = 1 \tag{8}$$

$$NS \cdot (z_{ij} + 1) + \sum_{s \in \mathcal{S}} (s - NS) \cdot x_{js} \geq \sum_{s \in \mathcal{S}} s \cdot x_{is} \qquad \forall i, j \in \mathcal{C} : \gamma_{ji} = 1 \tag{9}$$

$$x_{is} \in \{0, 1\} \qquad \forall i \in \mathcal{C}, \forall s \in \mathcal{S} \tag{10}$$

$$z_{ij} \in \{0, 1\} \qquad \forall i, j \in \mathcal{C} : \gamma_{ji} = 1 \tag{11}$$

$$y_{bw} \in \{0, 1\} \qquad \forall w \in \mathcal{W}, \forall b \in \mathcal{B}_w \tag{12}$$

Cost function (1) is the weighted sum of rehandling costs, costs associated with distances and costs of not loading containers. Constraints (2) impose that each container can be assigned at least to one slot, whereas (3) ensure that no more than one container is assigned to each slot. Constraints (4) guarantee that for each wagon only one weight configuration is chosen, while constraints (5), (6) and (7) are relative to the maximum weight for slots, wagons and trains, respectively. Constraints (8) and (9) impose the correct relation between the assignment variables x_{is} and the variables counting the rehandling operations z_{ij}. Finally, (10), (11) and (12) impose that the decision variables are binary.

2 Experimental tests and conclusions

We ran some experimental tests on a set of randomly generated problem instances in order to evaluate the effectiveness of the planning methodology described above. We used Cplex 12.1 to solve the 0-1 linear optimization problem and we adopted the ILOG Concert technology for building the model from the C♯ language. For space limitations, we cannot provide here many details on the computational tests but we simply give some general indications.

We considered real data (referred to a container terminal in the North of Italy) about the characteristics of wagons, the weight constraints, the storage area, the distances among the positions of containers in the storage area and the train slots, whereas we randomly generated the characteristics of containers (length, weight, priority, percentage of 20’ and

40' containers and so on) and the train composition. More specifically, containers were supposed to be stored along the railway tracks in two rows, up to the 4th tier; in each slot containers have the same length (20' or 40') and different commercial values. Moreover, we considered 5 types of wagons, either with a 2 TEU or 3 TEU capacity, 3 priority levels for containers, and the weight of containers varying randomly between 5 t and 30 t.

A preliminary experimental evaluation was devoted to a set of instances corresponding to the case of 300 containers in the stocking area and 4 trains composed of 30 wagons each, with 2 different destinations. Considering 10 instances of these dimensions, the solver was stopped after a time limit of 600 seconds, showing an optimality gap lower than 2%, on average, despite the large problem dimensions (280000 binary variables and 8000 constraints, on average). Because of the very low gap, we can conclude that with problem instances of such dimensions the mathematical programming formulation solved with Cplex is an effective way to realize the loading plan for multiple trains.

It is worth noting that the model described in this work is suitable for those real cases in which the loading plan for trains is realized when all the containers are ready in the stocking area and it is supposed that no new containers are brought to the stocking area while loading the trains. Of course, this situation can happen in some real cases, especially when the traffic volumes are not very high. Anyway, in many other cases, when a high number of trains per day must leave the terminal, it could be necessary to take into account in the problem formulation that the situation of the stocking area is dynamic.

The future research will be devoted to extend the proposed model formulation to take into account the dynamic evolution of the stocking area. To do that, it is necessary to consider the time explicitly in the model (i.e. the total time horizon can be discretized in some time steps); then, the storage position of containers is defined as a state variable and some state equations must be introduced in the model as constraints. In these state equations, of course, the position of the containers is updated depending on the train loading operations and the arrivals of new containers in the stocking area.

References

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