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Computational analysis of freeway traffic control based on a linearized prediction model

Lorenzo Maggi, Marco Maratea, Simona Sacone, Silvia Siri

Abstract— **This paper studies a control strategy for reducing congestions in freeway systems by applying ramp metering as control measure. More specifically, the objective is to define a Model Predictive Control scheme to be applied on line, characterized by a finite-horizon optimal control problem with a mixed-integer linear form, in order to be efficiently solved with commercial solvers. In this optimal control problem the prediction model is obtained by linearizing the first-order macroscopic traffic model, hence binary variables must be introduced and the resulting model has a piecewise linear structure. In the paper, the adopted control scheme is first of all analysed in order to evaluate its effectiveness in improving the traffic conditions; secondly, the analysis has been devoted to evaluate the computational time necessary to solve the finite-horizon optimal control problem depending on the problem sizes and the traffic scenarios.**

I. INTRODUCTION

The problem of traffic congestion in freeways has been addressed by researchers for some decades. Different traffic control measures have been proposed and implemented, such as ramp metering, variable speed limits, route guidance and vehicle-infrastructure integration systems [1]. The two most common control measures are ramp metering, which controls the traffic flow entering the freeway mainstream with traffic lights at on-ramps, and variable speed limits, which use on-road variable message signs to indicate specific speed limits. Ramp metering has been adopted successfully for more than 30 years, even though it presents limitations in some cases [2], [3]. The literature on ramp metering is very wide, starting from the feedback traffic controller ALINEA [4], until its extended versions such as the heuristic traffic-responsive feedback control strategy HERO [5] and the proportional-integral version PI-ALINEA [6]. On the other hand, variable speed limits are mainly used to control the driver speed of the mainline traffic [7], [8]. In many cases, variable speed limits and ramp metering are combined in order to improve the efficiency of the control scheme [9], [10].

The present paper is focused on the definition of a Model Predictive Control (MPC) scheme in order to reduce freeway congestion via ramp metering. In the last decades, many control approaches have been developed, until the most sophisticated non-linear MPC frameworks, as in [3], where the macroscopic non-linear second-order traffic model is adopted for the prediction. The major drawback of this approach is related to its computational burden: in fact, for each control interval, a non-linear programming problem must be solved and, then, the on-line application of this MPC scheme is possible only in small networks (as highlighted also in [11]).

Due to these computational problems of the non-linear MPC approach, some recent works have been devoted to find different solution techniques, as in [12], where an algorithm based on game theory is proposed. Other works (as for instance [13], [14]) propose MPC schemes for freeway ramp metering considering the Cell Transmission Model [15], [16] for the prediction. In these works, besides the non-linear formulation of the finite-horizon optimal control problem (FHOCP), the authors propose simpler formulations in which some model equations are relaxed.

In the present work we propose a control scheme in which the FHOCP is computationally affordable for an on-line application. To this end, we consider a simplified prediction model based on the first-order macroscopic dynamical model where the steady-state speed-density characteristic is discretized to become a piecewise constant function (as in [17]). In this way, we transform the classical non-linear optimization problem into a Mixed-Integer Linear Programming (MILP) problem, that can be solved efficiently by a MILP solver allowing to find the global optimum. Obviously, the piecewise linear prediction model is less accurate than the non-linear second-order model but it is shown with experimental tests that good performances of the proposed control law are assured and the computational strength is improved. In this paper we are especially interested in performing an experimental analysis about the computational effort needed to solve one single FHOCP, by generating random instances and analysing different traffic scenarios.

The idea of linearizing the prediction model in order to avoid the non-linear formulation has been also proposed in [18], where the second-order macroscopic traffic model is linearized, obtaining a more accurate prediction of the system dynamics, at the expense of a higher computational burden, with respect to the approach proposed in this paper. Besides, a similar approach is also provided in [19] where a mixed-integer formulation is adopted for the FHOCP in which, differently from the present approach, the linear prediction model is a linearized version of the Cell Transmission Model whereas the objective function has a quadratic form. In any case, the prediction models are restated in piecewise linear forms by introducing in the model some inequalities and some auxiliary variables, both binary and continuous, according to the definition of mixed logical dynamical systems [20].

The paper is organized as follows. In Section II the adopted (piecewise linear) prediction model is described

The authors are with the Department of Informatics, Bioengineering, Robotics and Systems Engineering, University of Genova, Italy

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whereas in Section III the finite-horizon optimal control problem is stated and analysed. The computational results are reported in Section IV and some conclusive remarks are drawn in Section V.

II. The piecewise linear prediction model

As already introduced, an MPC scheme is proposed in this paper. The considered MPC framework works as follows. At each time step k a FHOCP is solved over a prediction horizon K_p, by optimizing a suitable objective function subject to constraints on control variables and on state variables. The constraints also include the state equations, so that a prediction of the system behaviour is realized. A sequence of optimal control variables from time step k to $(k+K_p-1)$ is derived; the first element of this sequence becomes the control action at time step k. The same procedure is applied again at time step $k+1$ and iterated for all the following time steps.

In the proposed approach, the prediction model is a linearized version of the first-order macroscopic dynamical model of traffic flow, first introduced in the Fifties by Lighthill and Whitham [21], also called LW model. Of course, the LW model includes great simplifications if compared with the second-order one, proposed in the Seventies [22], [23] and applied in many real cases, as described for instance in [24]. Anyway, we adopt the first-order model in order to deal with a linear representation of the system and then to obtain a MILP formulation for the FHOCP.

Let us introduce the notation adopted for the macroscopic model considered in this work. First of all, $k=0,\dots,K-1$ denotes the temporal stage and $i=1,\dots,N$ indicates the section of the freeway stretch; T is the sample time interval and Δ_i is the length of section i. The main macroscopic variables, referred to section i and time step k, are the following:

- $\rho_i(k)$ is the traffic density [veh/km];
- $v_i(k)$ is the mean traffic speed [km/h];
- $q_i(k)$ is the traffic volume [veh/h];
- $l_i(k)$ is the queue length on the on-ramp [veh];
- $d_i(k)$ is the demand for access to the on-ramp [veh/h];
- $r_i(k)$ is the on-ramp traffic volume [veh/h];
- $s_i(k)$ is the off-ramp traffic volume [veh/h].

Note that if a certain section i is not provided with on-ramps and off-ramps, the corresponding variables $r_i(k)$, $s_i(k)$, $l_i(k)$ and $d_i(k)$ are imposed to be equal to 0. The first-order model can be written as follows:

$$\rho_i(k+1)=\rho_i(k)+\frac{T}{\Delta_i}\Big[q_{i-1}(k)-q_i(k)+r_i(k)-s_i(k)\Big] \quad (1)$$

$$l_i(k+1)=l_i(k)+T\big[d_i(k)-r_i(k)\big] \quad (2)$$

$$q_i(k)=\rho_i(k)\cdot V_f\cdot\exp\left[-\frac{1}{a_i}\left(\frac{\rho_i(k)}{\rho_i^{\mathrm{cr}}}\right)^{a_i}\right] \quad (3)$$

with $i=1,\dots,N$, $k=0,\dots,K-1$. Note that in the state equation for the traffic density (1), when considering the first section of the freeway stretch, corresponding to $i=1$, the term $q_{i-1}(k)$ represents the measured value of the traffic volume entering the freeway stretch. Equation (2) is the classical dynamic equation for the queues. Equation (3) implies that the traffic volume $q_i(k)$ adjusts instantaneously according to the fundamental diagram, in which V_f denotes the free speed (average speed assumed by vehicles when the traffic flows freely), ρ_i^{cr} is the critical density for section i (density at which the traffic flow is maximal), and a_i is a model parameter.

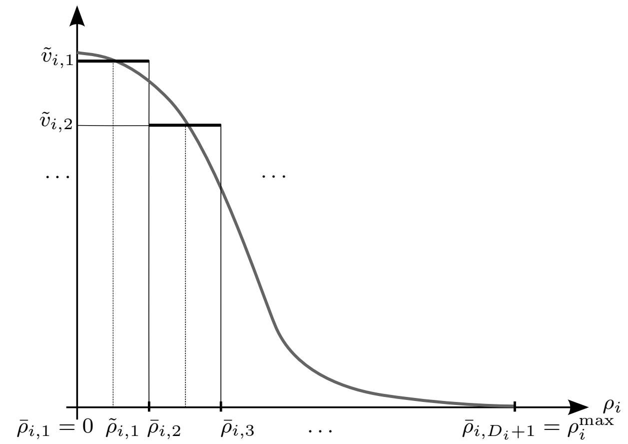

Fig. 1: The typical form of $V(\rho_i)$ and its piecewise constant approximation $\tilde{V}(\rho_i)$.

Our objective stands in further simplifying the model described above in order to obtain a linear formulation. To do that, the steady-state speed-density characteristic is simplified to become a piecewise constant function $\tilde{V}(\rho_i)$, $i=1,\dots,N$:

$$V(\rho_i)=V_f\cdot\exp\left[-\frac{1}{a_i}\left(\frac{\rho_i}{\rho_i^{\mathrm{cr}}}\right)^{a_i}\right]\approx\tilde{V}(\rho_i) \quad (4)$$

As shown in Figure 1, for each section i the range of variation of ρ_i is divided in D_i segments of equal length, with threshold values denoted as $\bar{\rho}_{i,j}$, $j=1,\dots,D_i$, with $\bar{\rho}_{i,1}=0$ and $\bar{\rho}_{i,D_i+1}=\rho_i^{\max}$. For each segment, the approximated values $\tilde{v}_{i,j}$ are obtained as the values of the speed-density characteristic computed in the average point of each discretization segment, i.e. in $\tilde{\rho}_{i,j}$. Now equation (1), taking into account (3), can be written as:

$$\rho_i(k+1)=\rho_i(k)+\frac{T}{\Delta_i}\Big[\tilde{\rho}_{i-1}(k)\tilde{v}_{i-1}(k) - \tilde{\rho}_i(k)\tilde{v}_i(k)+r_i(k)-s_i(k)\Big] \quad (5)$$

being $\tilde{\rho}_i(k)=\tilde{\rho}_{i,j}$ and $\tilde{v}_i(k)=\tilde{v}_{i,j}$, with $j:\bar{\rho}_{i,j}\le\rho_i(k)\le\bar{\rho}_{i,j+1}$, for $i=1,\dots,N$, $k=0,\dots,K-1$.

III. The finite-horizon optimal control problem

In order to express equations (5) in the FHOCP, it is necessary to identify the relevant discretization segment for each section i at time step k (on the basis of the value of the state variable $\rho_i(k)$). To this end, let us introduce some binary variables as follows:

$$y_{i,j}(k) = \begin{cases} 1 & \text{if } \rho_i(k) \geq \bar{\rho}_{i,j} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$z_{i,j}(k) = \begin{cases} 1 & \text{if } \rho_i(k) \leq \bar{\rho}_{i,j+1} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

with $i = 1, \ldots, N$, $j = 1, \ldots, D_i$, $k = 1, \ldots, K-1$. These definitions imply that, at time step k, the active discretization segment for section i is indicated by index j such that $y_{i,j}(k) = z_{i,j}(k) = 1$. Besides, another set of auxiliary variables must be introduced:

$$w_{i,j}(k) = \begin{cases} \tilde{v}_{i,j} & \text{if } \bar{\rho}_{i,j} \leq \rho_i(k) \leq \bar{\rho}_{i,j+1} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

with $i = 1, \ldots, N$, $j = 1, \ldots, D_i$, $k = 1, \ldots, K-1$.

The control objective includes the minimization of the Total Time Spent (TTS) by vehicles in the freeway system, as usually done in freeway optimal control problems. In addition, in order to make the control action more effective, a term is added to the TTS expression penalizing the state conditions in which the traffic density is higher than its critical value. To define such term of the cost function, it is necessary to introduce a further set of binary variables defined as:

$$x_i(k) = \begin{cases} 1 & \text{if } \rho_i(k) \geq \rho_i^{\text{cr}} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

with $i = 1, \ldots, N$, $k = 1, \ldots, K-1$.

It is now possible to state the FHOCP to be solved at time step k over a time horizon equal to K_p.

Problem 1: Given:

- the initial conditions on the queue length and the traffic volume $l_i(k)$, $q_i(k)$, $i = 1, \ldots, N$;
- the traffic volume entering the freeway stretch $q_0(h)$, $h = k, \ldots, K_p - 1$;
- the on-ramp demands and the off-ramp volumes $d_i(h)$, $s_i(h)$, $i = 1, \ldots, N$, $h = k, \ldots, K_p - 1$;

the problem is to find the optimal values of:

- the state variables $\rho_i(h)$ and $l_i(h)$, $i = 1, \ldots, N$, $h = k+1, \ldots, K_p$;
- the control variables $r_i(h)$, $i = 1, \ldots, N$, $h = k \ldots, K_p - 1$;
- the auxiliary variables $y_{i,j}(h)$, $w_{i,j}(h)$ and $z_{i,j}(h)$, $i = 1, \ldots, N$, $j = 1, \ldots, D_i$, $h = k+1, \ldots, K_p - 1$, and $x_i(h)$, $i = 1, \ldots, N$, $h = k+1, \ldots, K_p$;

minimizing the cost function:

$$\sum_{h=k+1}^{K_p} \sum_{i=1}^{N} c_1 T \Delta_i \rho_i(h) + c_2 T l_i(h) + c_3 x_i(h) \tag{10}$$

subject to

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\Delta_i}\Big[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)\Big] \qquad i = 1, \ldots, N \tag{11}$$

$$\rho_1(h+1) = \rho_1(h) + \frac{T}{\Delta_i}\Big[q_0(h) - \sum_{j=1}^{D_i} \tilde{\rho}_{1,j} w_{1,j}(h) + r_1(h) - s_1(h)\Big] \qquad h = k+1, \ldots, K_p - 1 \tag{12}$$

$$\rho_i(h+1) = \rho_i(h) + \frac{T}{\Delta_i}\Big[\sum_{j=1}^{D_i} \tilde{\rho}_{i-1,j} w_{i-1,j}(h) - \sum_{j=1}^{D_i} \tilde{\rho}_{i,j} w_{i,j}(h) + r_i(h) - s_i(h)\Big] \qquad i = 2, \ldots, N \quad h = k+1, \ldots, K_p - 1 \tag{13}$$

$$w_{i,j}(h) = \tilde{v}_{i,j} \cdot \Big(y_{i,j}(h) + z_{i,j}(h) - 1\Big) \quad i = 1, \ldots, N \quad j = 1, \ldots, D_i \quad h = k+1, \ldots, K_p - 1 \tag{14}$$

$$l_i(h+1) = l_i(h) + T\big[d_i(h) - r_i(h)\big] \qquad i = 1, \ldots, N \quad h = k, \ldots, K_p - 1 \tag{15}$$

$$\rho_i(h) - \bar{\rho}_{i,j} + M\big[1 - y_{i,j}(h)\big] > 0 \qquad i = 1, \ldots, N \quad j = 2, \ldots, D_i \quad h = k+1, \ldots, K_p - 1 \tag{16}$$

$$\bar{\rho}_{i,j} - \rho_i(h) + M y_{i,j}(h) \geq 0 \qquad i = 1, \ldots, N \quad j = 2, \ldots, D_i \quad h = k+1, \ldots, K_p - 1 \tag{17}$$

$$\rho_i(h) - \bar{\rho}_{i,j+1} + M z_{i,j}(h) > 0 \qquad i = 1, \ldots, N \quad j = 1, \ldots, D_i - 1 \quad h = k+1, \ldots, K_p - 1 \tag{18}$$

$$\bar{\rho}_{i,j+1} - \rho_i(h) + M\big[1 - z_{i,j}(h)\big] \geq 0 \qquad i = 1, \ldots, N \quad j = 1, \ldots, D_i - 1 \quad h = k+1, \ldots, K_p - 1 \tag{19}$$

$$\rho_i(h) - \rho_i^{\text{cr}} + M\big[1 - x_i(h)\big] > 0 \qquad i = 1, \ldots, N \quad h = k+1, \ldots, K_p \tag{20}$$

$$\rho_i^{\text{cr}} - \rho_i(h) + M x_i(h) \geq 0 \qquad i = 1, \ldots, N \quad h = k+1, \ldots, K_p \tag{21}$$

$$y_{i,1}(h) = 1, \; z_{i,D_i}(h) = 1 \qquad i = 1, \ldots, N, \quad h = k+1, \ldots, K_p - 1 \tag{22}$$

$$0 \leq \rho_i(h) \leq \rho_i^{\max} \qquad i = 1, \ldots, N \quad h = k+1, \ldots, K_p - 1 \tag{23}$$

$$0 \leq l_i(h) \leq l_i^{\max} \qquad i = 1, \ldots, N \quad h = k+1, \ldots, K_p - 1 \tag{24}$$

$$0 \leq r_i(h) \leq r_i^{\max} \qquad i = 1, \ldots, N \quad h = k \ldots, K_p - 1 \tag{25}$$

$$y_{i,j}(h) \in \{0,1\},\ z_{i,j}(h) \in \{0,1\} \qquad i = 1,\ldots,N$$
$$j = 1,\ldots,D_i \quad h = k+1,\ldots,K_p - 1 \quad (26)$$

$$x_i(h) \in \{0,1\} \qquad i = 1,\ldots,N \quad h = k+1,\ldots,K_p \quad (27)$$

where M is a sufficiently large number. □

In the cost function (10), c_1, c_2 and c_3 are weighting coefficients assigned to the three terms of the objective to be minimized (the first two referring to the computation of the TTS, the last one weighting when the critical density is exceeded). Constraints (11)-(13) represent the state equations on the traffic density, specifically defined for the first time step and for the first section. Constraints (14) express the definition of $w_{i,j}(h)$ variables, as in (8), while constraints (15) represent the dynamic equations on the queue lengths. Constraints (16)-(21) define the values of the binary variables $y_{i,j}(h)$, $z_{i,j}(h)$ and $x_i(h)$ as in (6), (7), (9), respectively. The other constraints define the lower and upper bounds for state, control and auxiliary variables.

It is worth pointing out that the FHOCP is a MILP that can be solved with commercial solvers; in the worst case, the computational time to optimally solve these types of problems exponentially depends on the number of binary variables of the formulation.

IV. Experimental evaluation

We have implemented the overall MPC scheme with the C♯ programming language; in particular, the FHOCP has been solved with the MILP solver Cplex 12.3 by using the ILOG Concert technology for building the model with the C♯ language. In order to test the performance of the proposed approach, we have adopted for the simulation the second-order macroscopic model presented in [7], [9] (with the same model parameters), that is an extended version of the METANET model [24]. All the experimental tests have been realized with a 2.53 GHz Intel(R) Core(TM) i5 computer with 4 GB RAM.

First of all, in order to evaluate the performance of the proposed ramp metering MPC approach, we have compared, in different traffic scenarios, the results of the no-control case with those obtained by applying the feedback traffic controller ALINEA and the MPC regulator proposed in this paper. For the experimental tests we consider a three-lane freeway stretch composed of $N = 7$ road sections of length Δ_i, $i = 1,\ldots 7$, equal to 1 kilometer, and a sample time T of 10 seconds. In the considered freeway stretch there are 3 on-ramps, in the first, the third and the fifth road section; an overall time horizon for the simulation corresponding to $K = 360$, i.e. 1 hour, is considered. For such system, a significant experimental campaign has been realized; although we discuss here one specific case, analogous conclusions can be drawn also for other traffic scenarios.

The analysed scenario corresponds to the case of a rather high density at the end of the considered stretch, equal to 160 [veh/km] from the first time step until $k = 250$ and equal to 70 [veh/km] in the remaining time steps; an inflow of 4800 [veh/h] for the whole time horizon has been considered; the

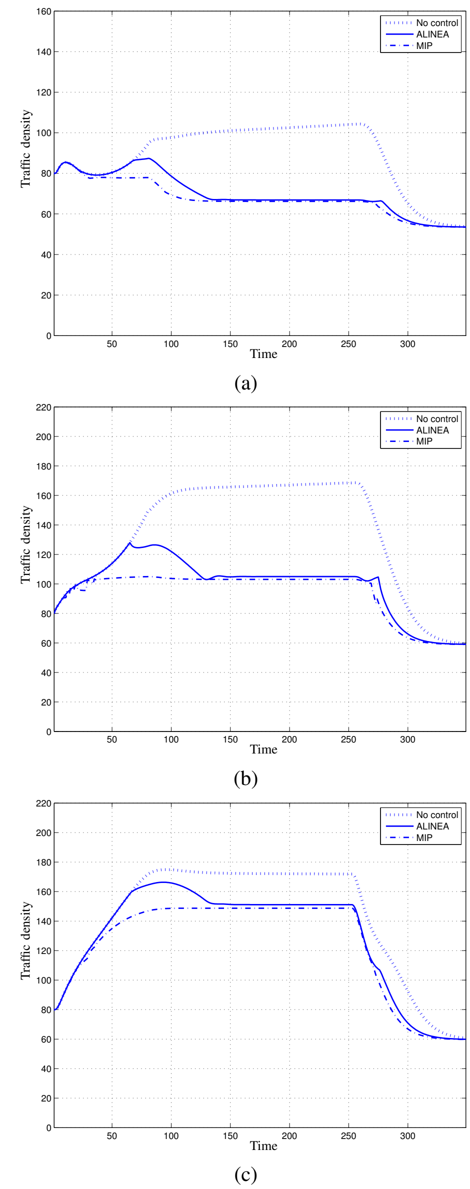

Fig. 2: Traffic density in section 4 (2a), 5 (2b) and 6 (2c).

initial density in all the road sections is equal to 80 [veh/km]. The demand at the three on-ramps is equal to 800 [veh/h] for the first 3 minutes, equal to 900 [veh/h] until minute 13 and then it is equal to 500 [veh/h].

We have applied the MPC regulator presented in this paper with a prediction horizon $K_p = 10$ and a discretization of the steady-state speed-density characteristic in $D_i = 12$ segments, $i = 1,\ldots 7$. With such parameters, each FHOCP is a MILP problem with about 2000 variables and more than 3000 constraints. Each problem is solved by Cplex in few seconds, that would be compatible with an on-line application, being the sample time equal to 10 seconds.

In Figure 2 it is possible to compare the different values of the traffic density in sections 4, 5 and 6, in case of no-control action, with the application of ALINEA (corresponding to a total TTS improvement of 4%) and with the application of the MPC scheme (involving an overall TTS improvement

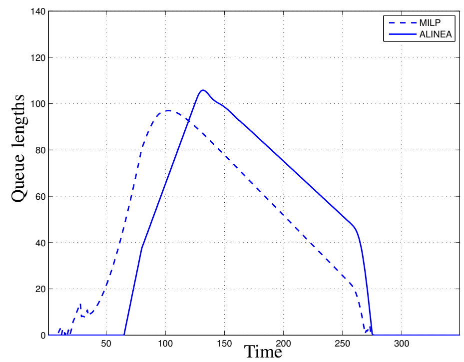

Fig. 3: Queue lengths in the on-ramp of section 5.

equal to 8%). It is easy to observe that the density in case of application of the MPC scheme is always lower than with ALINEA. Moreover, Figure 3 shows the queue lengths in the on-ramp of section 5, for the cases with ALINEA and MPC (the queues of the no-control case are not shown since they are always null). It is interesting to note that with the MPC scheme the queue length starts increasing before and this is due to the prediction model that is able to anticipate the higher on-ramp demand.

Once verified the good performance of the proposed control approach, a computational analysis has been performed, in order to evaluate two main aspects. First of all, it is of interest to analyse how the computational time changes when the sizes of the problem change (they are basically given by the number of freeway sections N, the prediction horizon length K_p and the number of segments in which the fundamental diagram is discretized D_i, $i = 1, \ldots, N$). Secondly, it is interesting to evaluate how the traffic conditions impact on the difficulty of the problem to be solved, i.e. on the computational time. In order to make a conservative analysis, we have randomly generated the traffic conditions in the freeway and we have solved different instances of the same problem type. It is also supposed that each freeway section has got an on-ramp, hence the number of control variables is the maximum possible number; this has been done, again, in order to perform a "worst case" analysis for the proposed control approach.

In order to examine the first point, i.e. the dependence of the problem computational burden on N, K_p and D_i, $i = 1, \ldots, N$, 8 groups of instances have been considered, as shown in Table I. Note that the number of discretization segments has been considered the same for the different sections and, from now on, denoted as D. For the second point, related to the analysis of different traffic conditions, we have taken into account two scenarios, whose main data are reported in Table II. The random data are the initial densities $\rho_i(k)$, $i = 1, \ldots, N$, the densities in the section before the first one $\rho_0(h)$, $h = k, \ldots, K_p - 1$, the demands at the on-ramps $d_i(h)$ and the exit flows $s_i(h)$, $i = 1, \ldots, N$, $h = k, \ldots, K_p - 1$. These values are randomly generated using the uniform distribution, as indicated in Table II. It is worth noting that scenario 1 represents regular traffic conditions in which the demands at the on-ramps are not very high and the initial and boundary conditions for the traffic density are always lower than the critical density. On the contrary, scenario 2 indicates a congested situation with higher demands and higher initial and boundary conditions for the traffic density.

TABLE I: The groups of instances

Group	N	K_p	D
1	5	7	10
2	5	7	12
3	7	7	10
4	7	7	12
5	7	10	10
6	7	10	12
7	10	10	10
8	10	10	12

TABLE II: Random data in the two scenarios

Scenario	$\rho_i(k)$	$\rho_0(h)$	$d_i(h)$	$s_i(h)$
1	U[70,90]	U[70,90]	U[1200,1600]	U[600,1000]
2	U[95,115]	U[95,115]	U[2200,2600]	U[1200,1600]

For each group and for each scenario, 5 random instances have been generated and solved, imposing a time limit to the solver equal to 60 seconds, since one minute can be considered as a maximum limit for an on-line application of the MPC scheme. Table III shows the computational results for scenario 1 reporting, for each group, the number of variables and constraints of the MILP problem, the average CPU time (in seconds) and the average optimality gap. The average CPU time is computed only over the instances optimally solved within the time limit (this number is reported in brackets in the same column), because for the other instances this is obviously equal to 60 seconds. Analogously, the average optimality gap is computed only over the instances not optimally solved, i.e. for which the solver has been stopped by the time limit (this number is reported in brackets), whereas for the others the optimality gap is obviously equal to 0. From Table III it can be seen that the instances of the first 4 groups are all optimally solved in few seconds, whereas for the other instances the optimal solution is not always reached. It can be noted that in group 5 three instances have been optimally solved whereas for the remaining two cases a high optimality gap is present. Group 6 is characterized by non-optimal solutions in all cases with a rather high optimality gap. Finally, as regards groups 7 and 8, the computational times strongly increase and for some instances one minute of computation is not sufficient.

TABLE III: Computational results for scenario 1

Group	Variables	Constraints	Avg. CPU Time	Avg. Opt. gap
1	1040	1575	0,302 (5)	-
2	1220	1875	3,314 (5)	-
3	1456	2205	0,431 (5)	-
4	1708	2625	4,914 (5)	-
5	2170	3297	1,944 (3)	59,27% (2)
6	2548	3927	-	28,611% (5)
7	3100	4710	18,118 (4)	99,997% (1)
8	3640	5610	28,574 (2)	99,926% (3)

The computational results for scenario 2 are reported in Table IV, where the columns have exactly the same meaning of Table III. It is immediately evident that, for the same problem sizes, the problem instances of scenario 2 are more difficult to be solved than those referred to scenario 1, showing how different traffic scenarios change the computational load of the problem to be solved. Hence, by remembering that we are working in the critical case in which each section has an on-ramp and random data are used, it is anyhow possible to conclude that considering a freeway stretch of 10 sections requires either a longer decision time interval or the definition of a decentralized control scheme in which smaller portions of the road are controlled separately.

TABLE IV: Computational results for scenario 2

Group	Variables	Constraints	Avg. CPU Time	Avg. Opt. gap
1	1040	1575	10,118 (5)	-
2	1220	1875	15,096 (4)	9,293% (1)
3	1456	2205	41,676 (4)	13,978% (1)
4	1708	2625	51,18 (1)	18,367% (4)
5	2170	3297	-	61,852% (5)
6	2548	3927	-	65,592% (5)
7	3100	4710	-	95,459% (5)
8	3640	5610	-	94,551% (5)

V. Conclusions

This paper has addressed the definition of an MPC scheme for reducing congestions in freeways by adopting ramp metering. This MPC framework adopts a piecewise linear version of the first-order macroscopic model for the prediction; the resulting finite-horizon optimal control problem has a mixed-integer linear form, hence it can be solved with MILP solvers.

In this paper the effectiveness of proposed MPC scheme for freeway traffic control has been tested via simulation. Moreover, further computational results regarding the solution of the FHOCP have been proposed. In particular, we have analysed the computational load of the FHOCP considering different problem sizes and different traffic conditions in the freeway to be controlled. The ultimate goal of such analysis is to determine the maximum problem sizes (in terms of number of sections, length of the prediction horizon and segments in which the fundamental diagram is discretized) for which the proposed control approach can be adopted on line in real cases. The experimental analysis, based on randomly generated instances, has shown that the proposed approach is suitable for medium traffic networks but, in case of larger networks, a decentralized approach could be more appropriate. Finally, as expected, the difficulty in solving the FHOCP strongly depends on the traffic scenario and, then, when validating a new scheme it is necessary to test it in different traffic scenarios, especially in the most congested cases.

References

[1] R. C. Carlson, I. Papamichail, M. Papageorgiou and A. Messmer, Optimal mainstream traffic flow control of large-scale motorway networks, *Transportation Research Part C*, vol. 18, 2010, pp 193-212.

[2] M. Papageorgiou, Freeway Ramp Metering: An Overview, *IEEE Transactions on Intelligent Transportation Systems*, vol. 3, 2002, pp 271-281.

[3] T. Bellemans, B. De Schutter, B. De Moor, Model predictive control for ramp metering of motorway traffic: A case study, *Control Engineering Practice*, vol. 14, 2006, pp 757-767.

[4] M. Papageorgiou, H. Hadj-Salem, J.-M. Blosseville, ALINEA: A local feedback control law for on-ramp metering, *Transportation Research Record*, vol. 1320, 1991, pp 58-64.

[5] I. Papamichail, E. Kosmatopoulos, M. Papageorgiou, I. Chrysoulakis, J. Gaffney and V. Vong, HERO coordinated ramp metering implemented at the Monash Freeway, in *Proc. of 2nd TRB International Symposium on Freeway and Tollway Operations*, 2009.

[6] Y. Wang, M. Papageorgiou, J. Gaffney, I. Papamichail, J. Guo, Local ramp metering in the presence of random-location bottlenecks downstream of a metered on-ramp, in *Proc. of 13th IEEE Conference on Intelligent Transportation Systems*, 2010, pp 1462-1467.

[7] A. Hegyi, B. De Schutter, H. Hellendoorn, Optimal coordination of variable speed limits to suppress shock waves, *IEEE Transactions on Intelligent Transportation Systems*, vol. 6, 2005, pp 102-112.

[8] M. Papageorgiou, E. Kosmatopoulos, I. Papamichail, Effects of variable speed limits on motorway traffic flow, *Transportation Research Record*, vol. 2047, 2008, pp 37-48.

[9] A. Hegyi, B. De Schutter, H. Hellendoorn, Model predictive control for optimal coordination of ramp metering and variable speed limits, *Transportation Research C*, vol. 13, 2005, pp 185-209.

[10] C. Caligaris, S. Sacone, S. Siri, Model predictive control for multiclass freeway traffic, *Proc. of the European Control Conference*, 2009, pp 1764-1769.

[11] J.R. Frejo and E.F. Camacho, Global versus local MPC algorithms in freeway traffic control with Ramp Metering and Variable Speed limits, *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, 2012, pp 1556-1565.

[12] A.H. Ghods, L. Fu, A. Rahimi-Kian, An efficient optimization approach to real-time coordinated and integrated freeway traffic control, *IEEE Transactions on Intelligent Transportation Systems*, vol. 11, 2012, pp 873-884.

[13] G. Gomes, R. Horowitz, Optimal freeway ramp metering using the asymmetric cell transmission model, *Transportation Research Part C*, vol. 14, 2006, pp 244-262.

[14] A. Muralidharan, R. Horowitz, Optimal control of freeway networks based on the Link Node Cell transmission model, *Proc. of American Control Conference*, 2012, pp 5769-5774.

[15] C. F. Daganzo, The Cell Transmission Model: A Dynamic Representation of Highway Traffic Consistent with the Hydrodynamic Theory, *Transportation Research B*, vol. 28, 1994, pp 269-287.

[16] C. F. Daganzo, The Cell Transmission Model, Part II: Network Traffic, *Transportation Research B*, vol. 29, 1995, pp 79-93.

[17] S. Sacone and S. Siri, A control scheme for freeway traffic systems based on hybrid automata, *Discrete Event Dynamic Systems: Theory and Applications*, vol. 22, 2012, pp 3-25.

[18] N. Groot, B. De Schutter, S. Zegeye, and H. Hellendoorn, Model-based predictive traffic control: A piecewise-affine approach based on METANET, *Proc. of 18th IFAC World Congress*, 2011, pp 10709-10714.

[19] A. Ferrara, A. Nai Oleari, S. Sacone, S. Siri, An event-triggered Model Predictive Control scheme for freeway systems, *Proc. of 51st IEEE Conference on Decision and Control*, 2012, pp 6975-6982.

[20] A. Bemporad and M. Morari, Control of systems integrating logic, dynamics, and constraints, *Automatica*, vol. 35, 1999, pp 407-427.

[21] M. J. Lighthill, J. B. Whitham, "On kinematic waves II: A theory of traffic flow on long crowded roads", in *Proc. of the Royal Society A*, vol. 229, pp. 317-345 (1955)

[22] H.J. Payne, "Models of freeway traffic and control", in *Mathematical Models of Public Systems, Simulation Council Proceedings*, pp. 51-61 (1971)

[23] G.B. Whitham, *Linear and Nonlinear Waves*. John Wiley, New York (1974)

[24] M. Papageorgiou, J.M. Blosseville, H. Hadj-Salem, Modelling and real-time control of traffic flow on the southern part of Boulevard Périphérique in Paris: Part I: Modelling, *Transportation Research Part A*, vol. 24, 1990, pp 345-359.