Solving Train Load Planning Problems with Boolean Optimization

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Context

- Transportation and logistics in seaport container terminals
- Train load planning problem: optimally assign containers to wagons of a train in order to satisfy capacity constraints while optimizing re-handling operations and train utilization
- Very important from the application view-point

Modeling aspects

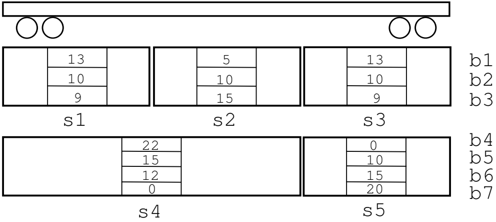

Table: Sketch of wagon weight restriction.

Hypothesis

- Wagons sequentially loaded (by cranes)
- Relative distance among containers and train is considered negligible
- One train at a time

Our contribution

- A mathematical model of the problem at hand
- that can be expressed as a PB, LP or SMT problem, and
- solved with Boolean Optimization solvers

Notation

- $\mathcal{C}$ is the set of containers; $\mathcal{W}$ is the set of wagons;
- ω_i is the weight of container $i \in \mathcal{C}$;
- π_i is the penalty paid if container i is not loaded, taking into account the urgency and commercial value of the container;
- $\gamma_{i,j}$, $i, j \in \mathcal{C}$, $i \neq j$, is the relative position between container i and j in the storage area; $\gamma_{i,j} = 1$ means that container i is located below j;
- $\overline{\Omega}_w$ and $\overline{\Omega}$ are the weight capacity of wagon $w \in \mathcal{W}$ and of the train;
- $\mathcal{S}$ is the set of slots; $\mathcal{S}_w$ is the set of possible slots for wagon w and, w_s indicates the wagon including slot s;
- $\mathcal{B}_w$ is the set of weight configurations for wagon w;
- $\delta_{b,s}$ is the maximum weight for slot s in the weight configuration b;
- α is the unitary re-handling cost in the storage area;
- T is the maximum number of containers stocked in a stack.

Decision variables

- $x_{i,s} \in \{0, 1\}$, equal to 1 if container i is assigned to slot s;
- $t_{w,b} \in \{0, 1\}$, equal to 1 if weight configuration b is chosen for wagon w;
- $y_{i,w} \in \{0, 1\}$, equal to 1 if container i is re-handled (i.e. it is moved but not assigned) when wagon w is loaded.

Constraints and optimization function

- Objective function minimizes
  - a term for re-handling operations, and
  - a term for the penalty of not loading containers
- Constraints express
  - that each container is assigned at most to one slot and that in each slot no more than one container is loaded
  - that for each wagon only one weight configuration is chosen
  - the maximum weight conditions for slots, wagons and the train
  - the re-handling operations

The basic model (I)

$$\min \ \alpha \cdot \sum_{i \in \mathcal{C}} \sum_{w \in \mathcal{W}} y_{i,w} + \sum_{i \in \mathcal{C}} \pi_i \cdot \left(1 - \sum_{s \in \mathcal{S}} x_{i,s} \right) \tag{1}$$

s.t.

$$\sum_{s \in \mathcal{S}} x_{i,s} \leq 1 \qquad \forall i \in \mathcal{C} \tag{2}$$

$$\sum_{i \in \mathcal{C}} x_{i,s} \leq 1 \qquad \forall s \in \mathcal{S} \tag{3}$$

$$\sum_{b \in \mathcal{B}_w} t_{w,b} = 1 \qquad \forall w \in \mathcal{W} \tag{4}$$

$$\sum_{i \in \mathcal{C}} \omega_i \cdot x_{i,s} \leq \sum_{b \in \mathcal{B}_{w_s}} \delta_{b,s} \cdot t_{w,b} \qquad \forall s \in \mathcal{S} \tag{5}$$

$$\sum_{i \in \mathcal{C}} \sum_{s \in \mathcal{S}_w} \omega_i \cdot x_{i,s} \leq \overline{\Omega}_w \qquad \forall w \in \mathcal{W} \tag{6}$$

The basic model (II)

$$\sum_{i \in \mathcal{C}} \sum_{s \in \mathcal{S}} \omega_i \cdot x_{i,s} \leq \overline{\Omega} \tag{7}$$

$$\sum_{j \in \mathcal{C}: \gamma_{j,i}=1} \sum_{s \in \mathcal{S}_w} x_{j,s} \leq (T-1) \cdot \left(y_{i,w} + \sum_{r \in \mathcal{S}_h : h < w} x_{i,r} \right) \qquad \forall i \in \mathcal{C} \quad \forall w \in \mathcal{W} \tag{8}$$

$$x_{i,s} \in \{0,1\} \qquad \forall i \in \mathcal{C} \quad \forall s \in \mathcal{S} \tag{9}$$

$$t_{w,b} \in \{0,1\} \qquad \forall w \in \mathcal{W} \qquad \forall b \in \mathcal{B}_w \tag{10}$$

$$y_{i,w} \in \{0,1\} \qquad \forall i \in \mathcal{C} \quad \forall w \in \mathcal{W} \tag{11}$$

The extended model

$y_{i,w}$ variables are associated with wagons but it could be more precise to associate these variables to slots: $z_{i,s}$, $s \in \mathcal{S}$.

$$\min\ \alpha \cdot \sum_{i \in \mathcal{C}} \sum_{s \in \mathcal{S}} z_{i,s} + \sum_{i \in \mathcal{C}} \pi_i \cdot \left(1 - \sum_{s \in \mathcal{S}} x_{i,s}\right) \qquad (12)$$

$$\sum_{j \in \mathcal{C}: \gamma_{j,i}=1} x_{j,s} \leq z_{i,s} + \sum_{r \in \mathcal{S}: r < s} x_{i,r} \qquad \forall i \in \mathcal{C} \quad \forall s \in \mathcal{S} \qquad (13)$$

in place of (1) and constraints (8).

Benchmarks and solvers evaluated

Benchmarks setting

- Some real world weights and restrictions (e.g. load configurations)
- Others data randomly generated

Table: Characteristics of the groups: mean of 10 instances.

Global setting			Basic model		Extended model	
Group	#containers	#wagons	#variables	#constraints	#variables	#constraints
A	20	10	941	345	1909	1337
B	30	10	1352	458	2884	2050
C	30	15	1977	670	4605	3151
D	40	15	2593	829	5855	4098

Solvers

- LP solvers: CPLEX
- PB solvers: WBO, PBCLASP, MINISAT+, BSOLO, GLPPB, SCIP
- SMT solvers: YICES, HYSAT

Results

Table: Basic and extended models: mean CPU time of solved instances and number of solved instances (in parenthesis) for CPLEX and SCIP. Time out of 1200s and memory limit of 500MB on a Linux box equipped with a Pentium IV 3.2GHz processor and 512MB of RAM.

Group	Basic model		Extended model	
	CPLEX	SCIP	CPLEX	SCIP
A	106.82(10)	116.86(8)	50.23(9)	164.17(9)
B	2.5(10)	75.9(10)	3.96 (10)	72.4(10)
C	68.47(8)	165.51(6)	230.42(3)	440.98(1)
D	15.37(9)	424.31(6)	285.23(6)	294.53(3)

Conclusions

- We have proposed a mathematical model to solve the train load planning problem
- We have generated a number of benchmarks based on this model
- solved with a number of Boolean Optimization solvers: the benchmarks
  - turn out to be generally difficult to solve
  - despite the relatively low size of the instances